Runaway Collapse of Witten Vortex Loops

Arne L. Larsen*[1] and Minos Axenides†[2]

February 7, 2008

[1] *Theoretical Physics Institute, Department of Physics, University of Alberta, Edmonton, Canada T6G 2J1*
[2] *Department of Physics, University of Crete, P.O. Box 2208, 71003 Iraklion, Greece*

Abstract

We consider general properties of charged circular cosmic strings in a general family of world-sheet string models. We then specialize to a model recently proposed by Carter and Peter. This model was shown to give a good description of the features of the superconducting cosmic strings, originally discovered by Witten in a $U(1)\times U(\tilde{1})$ field-theory. We derive an explicit expression for the potential determining the dynamics of the string and we present explicit expressions for the string tension and energy density, as a function of string-loop radius, in the locally preferred rest frame. We also obtain explicit expressions for the wiggle and woggle speeds (speeds of transverse and longitudinal perturbations, respectively). We show that the contraction of the uniformly charged string is essentially governed by the string tension (for large loop radius) and a *finite* Coulomb barrier (for small loop radius). We argue for the unobstructed contraction of a uniformly charged loop over the Coulombic barrier and its eventual collapse to a charged point. The implication of such an effect to the possible formation of naked singularities, in violation of the cosmic censorship hypothesis, is finally discussed.

PACS numbers: 0420, 1110, 1117

*Electronic address: alarsen@phys.ualberta.ca
†Electronic address: axenides@talos.cc.uch.gr

1 Introduction

Superconducting cosmic strings have attracted a lot of interest since the pioneering work by Witten [1]. Although charge-current carrying superconducting strings had been considered before [2] (see also [3]), the underlying spacetime field-theoretic explanation and understanding was missing. In a subsequent paper [4], it was proposed that static circular string-loops exist whose equilibrium configuration is characterized by an exact balance between the string tension and the current induced magnetic force (see also [5] for a discussion of static string-loops based on external magnetic fields). This idea initiated a series of papers on static string-loops [6-13], reaching a wide variety of conclusions. They nevertheless shared the same opinion that from a purely "mechanical" point of view, it is possible for the current to balance out the string tension; the only question was whether the current necessary for the balance would be below or above the maximum current J_{max} (the current at which the current-carrying ability saturates [1]) in realistic physical situations. Apparently the issue was settled in [12], concluding that the current necessary for balancing the string tension is indeed larger than J_{max}, that is, the current can not ensure the existence of static string-loops. However, in [12, 13] it was also proposed that a rotation of the superconducting string-loop might do the job of balancing the string tension (this is obviously impossible for an ordinary non-conducting Nambu-Goto string-loop); the resulting equilibrium configuration is thus strictly speaking stationary but not static. These results and ideas have later been confirmed and developed in a series of papers by Carter and Peter [14-19], for a review see Ref. [20] (chapter IV).

More or less independently, although inspired by these results, there has been some interest in the purely "mechanical" properties of non-rotating charge-current carrying superconducting string-loops. In these studies, the charge-current carrying superconducting string is described directly in terms of a two-dimensional world-sheet action, argued (or believed) to represent the effective action describing the dynamics of the core of a relatively thin, smooth and slowly varying 4-dimensional field-theoretic cosmic string, of the kind described by Witten [1]. Physical phenomena such as gravitational and electromagnetic radiation, the existence of a maximal superconducter current, backreaction effects on the background etc, are usually neglected in these papers.

The most popular of such world-sheet models is given by [21]:

$$\mathcal{S}_I = -\int d\tau d\sigma \sqrt{-G}\left(\mu + \frac{1}{2}G^{\alpha\beta}(\Phi_{,\alpha} + A_\mu X^\mu_{,\alpha})(\Phi_{,\beta} + A_\mu X^\mu_{,\beta})\right), \qquad (1.1)$$

where μ is the string tension, A_μ is the external electromagnetic potential, Φ is a

scalar field on the world-sheet and $G_{\alpha\beta}$ is the induced metric on the world-sheet:

$$G_{\alpha\beta} = g_{\mu\nu} X^{\mu}_{,\alpha} X^{\nu}_{,\beta}. \tag{1.2}$$

This model was used in most of the early papers on superconducting strings [6-8, 10-11, 21]. The purely mechanical properties of non-rotating string-loops described by the model (1.1), were investigated in [22-26]. Basically the result (in a flat Minkowski background devoid of external electromagnetic potentials) is that static circular string-loops can be obtained provided the string carries either a uniform charge density or a current (or both).

Another world-sheet model which has been discussed in detail in the literature is given by [2]:

$$\mathcal{S}_{II} = -\mu \int d\tau d\sigma \sqrt{-\left(1 + \frac{G^{\alpha\beta}}{\mu}(\Phi_{,\alpha} + A_\mu X^{\mu}_{,\alpha})(\Phi_{,\beta} + A_\mu X^{\mu}_{,\beta})\right) G} \tag{1.3}$$

The mechanical properties of non-rotating string-loops in this model ([27-30] as well as [22, 23, 25]) are somewhat different from the properties of the string-loops in the model (1.1). In particular, static string-loops can only be obtained provided the string carries *both* charge density *and* current [27]. It follows that a non-rotating string-loop with uniform charge density, but no current, has the somewhat unusual and unexpected (at least to us) property that it will contract and eventually collapse to a (charged) point, no matter how large the original charge density of the string is. This may have been part of the reason why most authors in the early years preferred the model (1.1), which seems to behave more like what is expected for a charged ring; namely, the contraction of a uniformly charged string-loop in the model (1.1) will stop before collapse due to an infinitely high Coulomb barrier.

It was therefore somewhat surprising when the numerical investigations [31] showed that in some respects, the model (1.3) gives a better description of Wittens field-theoretic cosmic string, than the model (1.1). Furthermore, a result of the numerical studies [31] was the recent proposal [32] of a new and mathematically somewhat more complicated model, which appearently gives a much better description of the Witten cosmic string than any of the models (1.1), (1.3).

The purpose of the present paper is to consider uniformly charged string-loops in this new world-sheet model by Carter and Peter [32].

The paper is organized as follows: In Section 2, we consider general properties of charged circular strings in a general family of world-sheet string models. We then specialize to the model recently proposed by Carter and Peter [32]. We derive an

explicit expression for the potential determining the dynamics of the string. In Section 3, we consider some physical properties of the string. In particular, we present explicit expressions for the string tension and energy density as a function of string-loop radius in the locally preferred rest frame, and we obtain explicit expressions for the wiggle and woggle speeds (speeds of transverse and longitudinal perturbations, respectively). We show that the contraction of a uniformly charged string in this model is essentially governed by the string tension (for large loop radius) and a *finite* Coulomb barrier (for small loop radius). We point out that a uniformly charged circular string can be above the *finite* Coulomb barrier and then collapse classically by simply contracting to a charged point. In Section 4, we give our conclusions and we present some speculations relating our results to the possible formation of naked singularities.

We use units where $4\pi\varepsilon_o = \hbar = c = 1$, such that $e \approx 1/137$ and $G \approx 2.1 \times 10^{15}(kg)^{-2}$. In these units, the string tension has dimension of mass-squared:

$$\mu \equiv m^2 \approx 10^{-21}(kg)^2, \tag{1.4}$$

where m is essentially the "Higgs" mass, and the numerical value is obtained for a GUT-string (see for instance [33]).

2 Charge-Current Carrying Strings

The starting point of our analysis will be a charge-current carrying string described by the following action [34]:

$$\mathcal{S} \;=\; \int d\tau d\sigma\; \mathcal{L}\,\sqrt{-G}, \tag{2.1}$$

where $\mathcal{L}$ is the Lagrangian density (being just a constant for the ordinary Nambu-Goto string). A very general family of string models that introduces electromagnetic self-interactions on the string as well as couplings to the external gravitational and electromagnetic potentials in a reparametrization invariant way, is obtained by letting the Lagrangian density $\mathcal{L}$ depend on the world-sheet projection of the gauge covariant derivative of a world-sheet scalar field Φ [34]:

$$\mathcal{L} = \mathcal{L}(\omega), \qquad \omega = G^{\alpha\beta}(\Phi_{,\alpha} + A_\mu X^\mu_{,\alpha})(\Phi_{,\beta} + A_\mu X^\mu_{,\beta}). \tag{2.2}$$

The spacetime energy-momentum tensor $T^{\mu\nu}$ and electromagnetic current J^μ are given by:

$$T^{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta\mathcal{S}}{\delta g_{\mu\nu}}, \tag{2.3}$$

$$J^{\mu} = \frac{1}{\sqrt{-g}} \frac{\delta \mathcal{S}}{\delta A_{\mu}}, \tag{2.4}$$

so that the total mass-energy M and charge Q of the string are:

$$M = \int d^3\vec{x}\, \sqrt{-g}\;\; T^{0}{}_{0}, \tag{2.5}$$

$$Q \;=\; \int d^3\vec{x}\, \sqrt{-g}\;\; J_0. \tag{2.6}$$

Explicit expressions for $T^{\mu\nu}$ and J^{μ} are:

$$\begin{aligned}\sqrt{-g}\, T^{\mu\nu} \;&=\; \int\; d\tau d\sigma\, \sqrt{-G}\;\; [\mathcal{L}\, G^{\alpha\beta} X^{\mu}_{,\alpha} X^{\nu}_{,\beta} - 2\frac{d\mathcal{L}}{d\omega} G^{\alpha\gamma} G^{\beta\delta} X^{\mu}_{,\gamma} X^{\nu}_{,\delta} (\Phi_{,\alpha} + A_{\rho} X^{\rho}_{,\alpha}) \\ &\qquad (\Phi_{,\beta} + A_{\sigma} X^{\sigma}_{,\beta})] \delta(X - X(\tau, \sigma)),\end{aligned} \tag{2.7}$$

and:

$$\sqrt{-g} J^{\mu} \;=\; 2\int d\tau d\sigma\, \sqrt{-G}\; \frac{d\mathcal{L}}{d\omega} G^{\alpha\beta} (\Phi_{,\alpha} + A_{\nu} X^{\nu}_{,\alpha}) X^{\mu}_{,\beta} \delta(X - X(\tau, \sigma)). \tag{2.8}$$

We will now specialize to circular strings in flat Minkowski space and zero external electromagnetic potential:

$$g_{\mu\nu} \;=\; \eta_{\mu\nu}, \qquad\qquad A_{\mu} = 0. \tag{2.9}$$

The circular strings are parametrized by:

$$t \;=\; E\tau, \qquad r \;=\; r(\tau), \qquad \theta \;=\; \frac{\pi}{2}, \qquad \phi \;=\; \sigma, \tag{2.10}$$

where E is a constant with dimension of length^{-1} (=mass) and $r(\tau)$ is to be determined from the equations of motion. We further consider Φ depending only on the world-sheet time τ:

$$\Phi \;=\; \Phi(\tau). \tag{2.11}$$

This is motivated by the fact that we will consider uniformly charged strings with zero current along the string. It turns out that eq.(2.11) precisely describes such strings.

Let us now consider the equations of motion as obtained from eqs.(2.1)-(2.2) for the circular strings, eqs.(2.10)-(2.11). It is straightforward to show that these equations lead to [35]:

$$\dot{r}^2 \;=\; E^2 \;-\; r^2 \left(\mathcal{L} + \frac{\Omega^2}{2r^2 (d\mathcal{L}/d\omega)} \right)^2, \tag{2.12}$$

$$\dot{\Phi} = \frac{\Omega\sqrt{E^2-\dot{r}^2}}{2r(d\mathcal{L}/d\omega)}, \tag{2.13}$$

$$\omega = \frac{-\Omega^2}{4r^2(d\mathcal{L}/d\omega)^2}\ , \tag{2.14}$$

where Ω is a dimensionless integration constant. It follows that:

$$\sqrt{-g}\ J_0 = E\Omega \int d\tau d\sigma\ \delta(X - X(\tau,\sigma)), \tag{2.15}$$

$$\sqrt{-g}\ T^0_{\ 0} = E^2 \int d\tau d\sigma\ \delta(X - X(\tau,\sigma)). \tag{2.16}$$

The charge and mass-energy of the string are then given by:

$$Q = E\Omega \int d\tau d\sigma\ \delta(t - t(\tau)) = 2\pi\Omega, \tag{2.17}$$

$$M = E^2 \int d\tau d\sigma\ \delta(t - t(\tau)) = 2\pi E, \tag{2.18}$$

so that (E, Ω) are the energy and charge densities of the string, respectively. Notice also that all these results have been obtained without specifying $\mathcal{L}(\omega)$ at all !

Different Lagrangians $\mathcal{L}$ essentially correspond to different ways of introducing charge (and current) on the string. The two most popular models studied in the literature are (c.f. eqs.(1.1), (1.3)):

$$\mathcal{L}_I = -(\mu + \frac{\omega}{2}), \tag{2.19}$$

$$\mathcal{L}_{II} = -\mu\sqrt{1+\frac{\omega}{\mu}}\ . \tag{2.20}$$

For these models one can easily solve eq.(2.14) for $\omega = \omega(\Omega^2, r^2, \mu)$ and then eq.(2.12) becomes:

$$\dot{r}^2 + V(r) = E^2, \tag{2.21}$$

where:

$$V_I = (\mu r + \frac{\Omega^2}{2r})^2, \tag{2.22}$$

$$V_{II} = \mu(\mu r^2 + \Omega^2), \tag{2.23}$$

determining the dynamics of the string. These two models have been discussed extensively in the literature [22-30], so we shall not repeat the results here. Furthermore, it was recently shown by Carter and Peter [32] that a much more realistic model is provided by:

$$\mathcal{L}_{III} = -\ \mu\ -\frac{\omega}{2}(1+\frac{\omega}{m_*^2})^{-1}\ , \tag{2.24}$$

which, besides the string tension $\mu = m^2$, depends on an additional parameter m_*^2. It was shown [32] that the model of eq.(2.24) gives a much better description of the original Witten vortex [1], than do the previously presented models of eqs.(2.19) – (2.20). In the present paper we will not address to this issue any further. We will merely take the model of eq.(2.24) as it stands and consider the dynamics of circular strings as derived from it.

In the case of $\mathcal{L}_{III}$, eq.(2.14) becomes:

$$\Omega^2 (1 + \frac{\omega}{m_*^2})^4 + \omega r^2 = 0, \tag{2.25}$$

to be solved for $\omega = \omega(r^2, \Omega^2, m_*^2)$. Introduce the two dimensionless quantities:

$$x \equiv |\frac{m_*}{\Omega}| r \geq 0, \qquad v \equiv -\frac{\omega}{m_*^2}. \tag{2.26}$$

Then eq.(2.25) has two real solutions given by:

$$v_\pm = 1 + \frac{1}{2}\sqrt{W(x)} \pm \frac{1}{2}\sqrt{-W(x) + \frac{2x^2}{\sqrt{W(x)}}}\ , \tag{2.27}$$

where:

$$W(x) \ \equiv \ \frac{Z(x)}{54^{1/3}} - \frac{128^{1/3} x^2}{Z(x)}\ , \tag{2.28}$$

$$Z(x) \ \equiv \ \left(27x^4 + 3\sqrt{3}\sqrt{x^6(256 + 27x^4)}\ \right)^{\frac{1}{3}}. \tag{2.29}$$

The potential in eq.(2.21) now takes the form (in dimensionless quantities):

$$\frac{1}{\Omega^2 m_*^2} V_\pm = x^2 \left(\frac{\mu}{m_*^2} - \frac{v_\pm}{2}(1 - v_\pm)^{-1} + \frac{1}{x^2}(1 - v_\pm)^2 \right)^2. \tag{2.30}$$

This expression, once written out completely using eqs.(2.26)-(2.29), gives the potential explicitly and analytically as a function of the string-loop radius, but it is not very enlightening. It is more useful to consider the two asymptotic regions $x \to 0$ $(r \to 0)$ and $x \to \infty$ $(r \to \infty)$. For $x \to 0$, eq.(2.27) has the expansion:

$$v = \sum_{i=0}^{\infty} a_i x^{i/2}, \tag{2.31}$$

where:

$$a_i = \sum_{j=1}^{i+1} \sum_{k=1}^{i+2-j} \sum_{l=1}^{i-j-k+3} a_{4+i-j-k-l}\, a_j\, a_k\, a_l, \qquad a_0 = 1. \tag{2.32}$$

That is to say, $a_1 = \pm 1, \quad a_2 = \frac{1}{4}, \quad a_3 = \mp\frac{1}{32}$ etc. It follows that:

$$v_\pm = 1 \pm \sqrt{x} + \frac{x}{4} \mp \frac{1}{32} x^{3/2} + \mathcal{O}(x^2). \tag{2.33}$$

The potential (2.30) then is:

$$\frac{1}{\Omega^2 m_*^2} V_\pm = 1 \pm 2\sqrt{x} + \mathcal{O}(x), \tag{2.34}$$

for $x \longrightarrow 0$, i.e. :

$$V_\pm(0) = \Omega^2 m_*^2, \qquad V'_\pm(0) = \pm\infty, \tag{2.35}$$

where prime denotes derivative with respect to x.

For $x \longrightarrow \infty$, we find instead:

$$v_+ \quad = \quad x^{\frac{2}{3}} + \mathcal{O}(1), \tag{2.36}$$

$$v_- \quad = \quad x^{-2} + \mathcal{O}(x^{-4}), \tag{2.37}$$

and then the potential (2.30) is :

$$\frac{1}{\Omega^2 m_*^2} V_+ = (\frac{\mu}{m_*^2} + \frac{1}{2})^2 x^2 + \mathcal{O}(x^{2/3}), \tag{2.38}$$

$$\frac{1}{\Omega^2 m_*^2} V_- = (\frac{\mu}{m_*^2})^2 x^2 + \mathcal{O}(1), \tag{2.39}$$

for $x \longrightarrow \infty$, i.e. :

$$V_\pm(\infty) = \infty. \tag{2.40}$$

Notice that the results obtained for the potential V_+ are somewhat similar (although there are small differences) to results obtained for the model (2.19) of Nielsen [22, 23, 25, 27-30], while (2.20) gives completely different results [22-26], especially for $r \longrightarrow 0$. The results obtained for the potential V_- differs from the results of both models (2.19)-(2.20), especially for $r \longrightarrow 0$.

For the model (2.24), we have now shown that in both cases ($V_\pm$) the potential goes to infinity at spatial infinity but it takes a finite value at $x = 0$ ($r = 0$). So

there is no infinite Coulomb barrier for these charged strings. This is confirmed by numerical plots of $V_{\pm}$, Fig.1. It follows that a uniformly charged circular string in the model (2.24) will collapse provided:

$$E^2 \geq V_{\pm}(0) \ = \ \Omega^2 m_*^2. \tag{2.41}$$

That is, if (2.41) is fulfilled, a charged string-loop will collapse classically by simply contracting to a charged point.

In the next section we turn to the physical interpretation of the above results. Are they merely artifacts of an unphysical model or do they actually describe real physics ?

3 The Physical Interpretation

In this section we will consider the physical interpretation of the results obtained in Section 2. In particular we address the question of whether they are trustworthy or whether they have been simply obtained through an extrapolation of the model (2.24) outside its range of validity.

In the notation of Ref. [32], the Lagrangian (2.24) reads:

$$\tilde{\Lambda} \ = \ -m^2 + \frac{\tilde{\chi}}{2}(1 - \frac{\tilde{\chi}}{m_*^2})^{-1} \ , \tag{3.1}$$

that is:

$$\omega = -\tilde{\chi} = -m_*^2 v. \tag{3.2}$$

Since v is positive for the circular strings, we are always in the "Electric" range [32]. The energy density U and string tension T, in the locally preferred rest frame, are then given by [32]:

$$U \ = \ m^2 - \frac{\tilde{\chi}}{2}(1 - \frac{\tilde{\chi}}{m_*^2})^{-1} + \tilde{\chi}(1 - \frac{\tilde{\chi}}{m_*^2})^{-2} \ , \tag{3.3}$$

$$T \ = \ m^2 - \frac{\tilde{\chi}}{2}(1 - \frac{\tilde{\chi}}{m_*^2})^{-1} \ . \tag{3.4}$$

The wiggle and woggle speeds (i.e., the speeds of transverse and longitudinal perturbations, respectively) are [32]:

$$c_E^2 \ = \ \frac{T}{U} \ = \ 1 - \frac{\tilde{\chi}(1 - \frac{\tilde{\chi}}{m_*^2})^{-2}}{m^2 - \frac{\tilde{\chi}}{2}(1 - \frac{\tilde{\chi}}{m_*^2})^{-1} + \tilde{\chi}(1 - \frac{\tilde{\chi}}{m_*^2})^{-2}} \ , \tag{3.5}$$

$$c_L^2 = -\frac{dT}{dU} \; = \; \frac{1 - \frac{\tilde{\chi}}{m_*^2}}{1 + \frac{3\tilde{\chi}}{m_*^2}} \;\; . \tag{3.6}$$

Notice that by using eqs.(2.26)-(2.29), U, T, c_E and c_L are given explicitly and analytically as functions of the string-loop radius. These expressions hold when $\tilde{\chi} > 0$. However in order to ensure that $c_E^2 > 0$, $c_L^2 > 0$ as well as $c_L < c_E < 1$ (the condition $c_L < c_E$ must be fulfilled for the model (2.24) to describe properly the Witten vortex [32]), we must further have [32]:

$$0 < \frac{\tilde{\chi}}{m_*^2} \; < \; 1 \; - \; \frac{3m_*^2}{2(2m^2 + m_*^2)} \;\; . \tag{3.7}$$

In our notation, eq.(3.7) reads:

$$0 < v_\pm < 1 - \frac{3m_*^2}{2(2\mu + m_*^2)} \;\; . \tag{3.8}$$

This condition is certainly not fulfilled in general for our solutions (2.27). In fact, $v_+ \geq 1$ so that the corresponding string solution must be neglected as being unphysical, since it actually has an imaginary woggle speed c_L. On the other hand, $v_- \leq 1$ and it is therefore possible that eq.(3.8) gets fulfilled for the corresponding string solution. In Fig.2 we plot (c_E, c_L) as a function of the radius of this string-loop. Clearly $0 \leq c_L < c_E \leq 1$, except for $x \rightarrow 0$. It means that the circular string, which is determined by eq.(2.21) with the potential V_- given by eq.(2.30) is supposed to be a good approximation to a circular Witten vortex for any loop radius larger than some r_0. Moreover, by increasing m^2/m_*^2 we can make this r_0 arbitrarily small. Physically we actually expect $m^2/m_*^2 >> 1$, since m_*^2 represents the charge-carrier mass while m^2 represents the Higgs mass [32].

In conclusion, the circular string solution corresponding to the potential V_- can essentially be trusted everywhere. Therefore, a uniformly charged circular string that fulfills eq.(2.41) collapses classically to a charged point, as there is no Coulomb barrier preventing the collapse.

4 Conclusion and Discussion

We have considered charged circular cosmic strings in a world-sheet model recently proposed by Carter and Peter [32]. We derived an explicit expression for the potential determining the dynamics of the string, and we presented explicit expressions for the string tension, the energy density and the wiggle and woggle speeds, as a function of the string-loop radius. We showed that a uniformly charged circular

string in this model can be above the *finite* Coulomb barrier and then collapse classically by simply contracting to a charged point. This suggests that the end-result of contraction will be a Reissner-Nordstrom black hole (the end-result of contraction of an uncharged circular string is a Schwarzschild black hole). However, we shall now argue for the possibility (at least in principle) of the formation of a naked singularity. After the collapse, we expect that spacetime is described by the Reissner-Nordstrom line-element:

$$ds^2 = -a(r)dt^2 + \frac{dr^2}{a(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{4.1}$$

where in our units:

$$a(r) = 1 - \frac{2GM}{r} + \frac{GQ^2}{r^2}. \tag{4.2}$$

This line-element describes a naked singularity if:

$$GM^2 < Q^2, \tag{4.3}$$

where $M = 2\pi E$ and $Q = 2\pi\Omega$. For ordinary matter it is usually expected that if the initial conditions are such that eq.(4.3) holds, then the Coulomb barrier will prevent the collapse from taking place (cosmic censorship), thus the naked singularity will not form. Interestingly enough, this does not seem to be the case for the charged cosmic string considered in this paper (which is supposed to be a good approximation to the original field-theoretic Witten-vortex). For the charged string to actually collapse, we have the restriction (2.41), but eqs.(2.41), (4.3) are actually fully consistent if the following relation holds:

$$Gm_*^2 < 1. \tag{4.4}$$

Now since $m_*^2 << m^2$, we have that $Gm_*^2 << 1$ when using the numerical values for a GUT-string (1.4), so eq.(4.4) is trivially fulfilled. This suggests the possibility of formation of a naked singularity if the initial charged circular cosmic string is prepared such that eq.(4.3) holds. It should be stressed, however, that we have completely neglected such physical effects as gravitational and electromagnetic radiation, backreaction etc, which might change the above conclusions (for instance as in [36]). We hope to return to these questions elsewhere.

Acknowledgements

The research of M.A. was partially supported by Danmarks Grund-Forsknings Fond through its support for the establishment of the Theoretical Astrophysics Center, while A.L.L. was supported by NSERC (National Sciences and Engineering Research Council of Canada). A.L.L. also thanks B. Carter for discussions on the results of Ref. [32], prior to its publication.

References

[1] E. Witten, Nucl. Phys. B249 (1985) 557.

[2] N.K. Nielsen, Nucl. Phys. B167 (1980) 248.

[3] A.P. Balachandran, B.S. Skagerstam and A. Stern, Phys. Rev. D20 (1979) 439.

[4] J.P. Ostriker, C. Thompson and E. Witten, Phys. Lett. B180 (1986) 231.

[5] N.K. Nielsen and P. Olesen, Nucl. Phys. B291 (1987) 829; Nucl. Phys. B298 (1988) 776.

[6] E. Copeland, M. Hindmarsh and N. Turok, Phys. Rev. Lett. 58 (1987) 1910.

[7] D.N. Spergel, T. Piran and J. Goodman, Nucl. Phys. B291 (1987) 847.

[8] C.T. Hill, H.M. Hodges and M.S. Turner, Phys. Rev. D37 (1987) 263; Phys. Rev. Lett. 59 (1987) 2493.

[9] A. Babul, T. Piran and D.N. Spergel, Phys. Lett. 202 (1988) 307.

[10] E. Copeland, D. Haws, M.Hindmarsh and N. Turok, Nucl. Phys. B306 (1988) 908.

[11] D. Haws, M. Hindmarsh and N. Turok, Phys. Lett. B209 (1988) 255.

[12] R.L. Davis and E.P.S. Shellard, Phys. Lett. B209 (1988) 485.

[13] R.L. Davis and E.P.S. Shellard, Nucl. Phys. B323 (1989) 209.

[14] B. Carter, in "Early Universe and Cosmic Structures", eds. A. Blanchard and J. Tran Thanh Van (Editions Frontieres, Gif-Sur-Yvette, 1990).

[15] B. Carter, Phys. Lett. B238 (1990) 166.

[16] B. Carter, in "Formation and Evolution of Cosmic Strings", eds. G. Gibbons, S. Hawking and T. Vachaspati (Cambridge University Press, 1990).

[17] B. Carter, Ann. N.Y. Acad. Sci. 647 (1991) 758.

[18] P. Peter, Phys. Rev. D46 (1992) 3322.

[19] P. Peter, Phys. Lett. B298 (1993) 60.

[20] B. Carter, in "Formation and Interactions of Topological Defects", ed. R. Brandenberger and A-C. Davis (Plenum Press, N.Y., 1995).

[21] A. Vilenkin and T. Vachaspati, Phys. Rev. Lett. 58 (1987) 1041.

[22] A. Davidson and K.C. Wali, Nucl. Phys. B349 (1991) 581.

[23] A.L. Larsen, Phys. Lett. A170 (1992) 174.

[24] A. Davidson, N.K. Nielsen and Y. Verbin, Nucl. Phys. B412 (1994) 391.

[25] B. Boisseau, H. Giacomini and D. Polarski, Phys. Rev. D51 (1995) 6909.

[26] M. Axenides and A.L. Larsen, Int. J. Mod. Phys. D4 (1995) 723.

[27] A. Davidson and K.C. Wali, Phys. Lett. B213 (1988) 439.

[28] A. Davidson and R. Zamir, Phys. Lett. B223 (1989) 189.

[29] A. Davidson, Phys. Rev. Lett. 65 (1990) 145.

[30] A.L. Larsen, Phys. Lett. B273 (1991) 375; Mod. Phys. Lett. A7 (1992) 2913.

[31] P. Peter, Phys. Rev. D45 (1991) 1091; Phys. Rev. D47 (1993) 3169.

[32] B. Carter and P. Peter, Phys. Rev. D52 (1995) 1744.

[33] A. Vilenkin, Phys. Rep. 121 (1985) 263.

[34] B. Carter, Phys. Lett B224 (1989) 61; Phys. Lett. B228 (1989) 466.

[35] A. L. Larsen, Class. Quantum Grav. 10 (1993) 1541.

[36] C. Vaz and L. Witten, "Do Naked Singularities Form", Preprint UATP-95/03.

Figure Captions

Fig.1. The potential (2.30) determining the dynamics of a charged string-loop. Notice that only V_- shows a Coulomb barrier, which furthermore is finite.

Fig.2. The wiggle and woggle speeds (3.5)-(3.6), as a function of string-loop radius, for the string corresponding to the potential V_-. Clearly $0 \leq c_L < c_E \leq 1$, except for $r \to 0$.